Tangensov poučak

Armando Slaviček

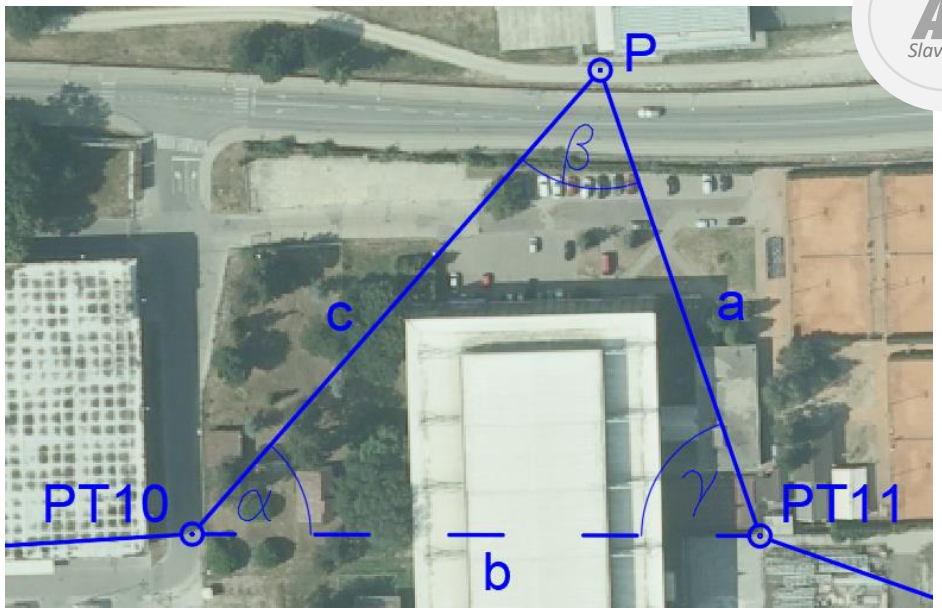
Tangensov poučak

$$\frac{a+b}{a-b}=\frac{\tan(\frac{\alpha+\beta}{2})}{\tan(\frac{\alpha-\beta}{2})}$$

$$\frac{b+c}{b-c}=\frac{\tan(\frac{\beta+\gamma}{2})}{\tan(\frac{\beta-\gamma}{2})}$$

$$\frac{c+a}{c-a}=\frac{\tan(\frac{\gamma+\alpha}{2})}{\tan(\frac{\gamma-\alpha}{2})}$$

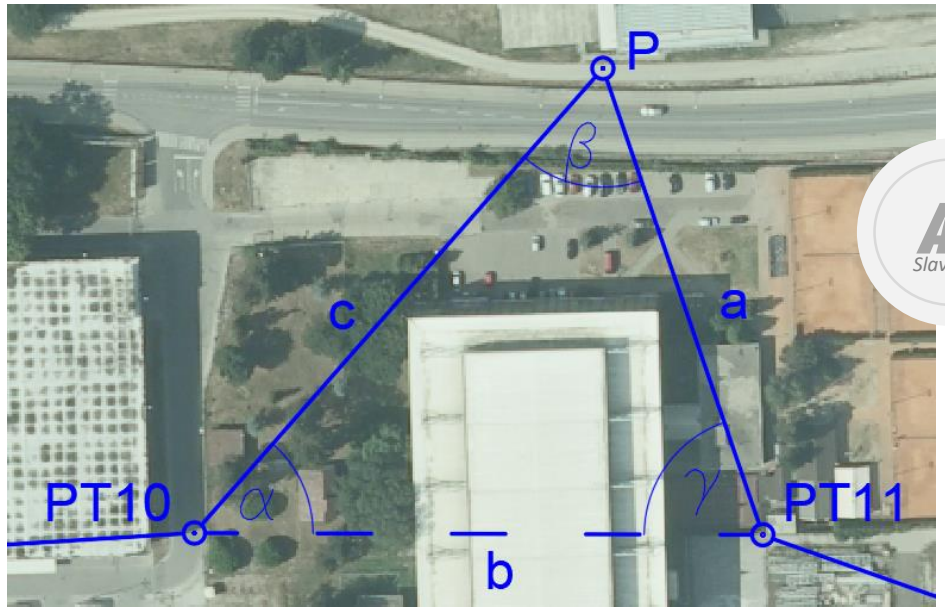
Omjer zbroja i razlike duljina dviju stranica trokuta jednak je omjeru tangensa poluzbroja i polurazlike nasuprotnih kutova.



Funkcija tangens i funkcija kosinus koriste se kada su poznate dvije stranice i kut između njih.

Tangensov poučak ćemo koristiti kada se između krajnjih točaka poligonske stranice, koju treba izmjeriti, nalazi prepreka koja onemogućava da se točke međusobno dogledaju.
Pomoću pomoćne točke P stvorit ćemo trokut PT10-P-PT11 i izmjeriti kut β i dvije stranice a i c.

Tangensov poučak



$$\frac{a+c}{a-c}=\frac{\tan(\frac{\alpha+\gamma}{2})}{\tan(\frac{\alpha-\gamma}{2})}$$

α+γ = 180° - β ➡ $\frac{\alpha+\gamma}{2}=90°-\frac{\beta}{2}$

$$tan\left(\frac{\alpha+\gamma}{2}\right)=tan\left(90°-\frac{\beta}{2}\right)=cot\frac{\beta}{2}$$

$$\frac{a+c}{a-c}=\frac{\tan(\frac{\alpha+\gamma}{2})}{\tan(\frac{\alpha-\gamma}{2})} \Rightarrow \tan(\frac{\alpha-\gamma}{2})=\frac{a-c}{a+c}\tan(\frac{\alpha+\gamma}{2}) \Rightarrow \tan(\frac{\alpha-\gamma}{2})=\frac{a-c}{a+c}cot(\frac{\beta}{2})$$

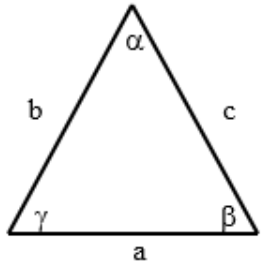
Računanje trokuta iz dviju mjerenih duljina stranica i kuta između njih

Tangensov poučak

Trigonometrijski obrazac br. 14 - tan

Računanje trokuta iz dviju mjerenih duljina stranica i kuta između njih

Skica



Mjerene veličine
α, b, c

Računate veličine
a, β, γ

Kontrola
α+β+γ = 180°

$$\frac{(\beta+\gamma)}{2} = 90° - \frac{\alpha}{2}$$

$$tan\frac{(\beta-\gamma)}{2} = \frac{b-c}{b+c}\cot\frac{\alpha}{2}$$

6 $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$$\beta = \frac{\beta+\gamma}{2} + \frac{\beta-\gamma}{2}$$

5 $$\gamma = \frac{\beta+\gamma}{2} - \frac{\beta-\gamma}{2}$$

7 * $a = (b/\sin\beta)\sin\alpha$
$a = (c/\sin\gamma)\sin\alpha$

AS

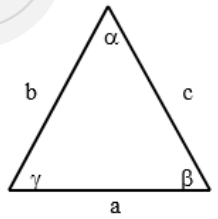
		°	'	"					ASlaviček
arctan = inverzna funkciji tangensa	α	59	28	52	b	191,53		1 α/2	29,74055556
	β	62	25	10	c	183,45	B	3 cot (α/2)	1,750306753
5	γ	58	5	58	6 a	186,150		b-c	8,08
$\frac{(\beta+\gamma)}{2} = 90° - \frac{\alpha}{2}$	Σ	180	00	00				b+c	374,98
2	(β+γ)/2	60	15	34	Kontrola*		A	(b-c)/(b+c)	0,021547816
$tan\frac{(\beta-\gamma)}{2} = \frac{b-c}{b+c}\cot\frac{\alpha}{2}$ 4	(β-γ)/2	2	9	36	7 a	186,150		4 arctan (A * B)	2,1599031

Računanje trokuta iz dviju mjerenih duljina stranica i kuta između njih

Tangensov poučak

Trigonometrijski obrazac br. 14 - tan

Računanje trokuta iz dviju mjerenih duljina stranica i kuta između njih



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$tan\frac{(\beta-\gamma)}{2} = \frac{b-c}{b+c}\cot\frac{\alpha}{2}$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\beta = \frac{\beta+\gamma}{2} + \frac{\beta-\gamma}{2}$

$\gamma = \frac{\beta+\gamma}{2} - \frac{\beta-\gamma}{2}$

* a = (b/sin β) sin α
a = (c/sin γ) sin α

AS | +/- | ° | ' | " | | | | | ASlaviček

arctan = inverzna funkciji tangensa

$\frac{(\beta+\gamma)}{2} = 90° - \frac{\alpha}{2}$

$tan\frac{(\beta-\gamma)}{2} = \frac{b-c}{b+c}\cot\frac{\alpha}{2}$

α		51	34	42	b	171,65		α/2
β					c	169,25	B	cot (α/2)
γ					a			*b-c*
Σ								*b+c*
(β+γ)/2					*Kontrola**		A	*(b-c)/(b+c)*
(β-γ)/2					*a*			*arctan (A * B)*
α		51	31	65	b	169,26		α/2
β					c	167,58	B	cot (α/2)
γ					a			*b-c*
Σ								*b+c*
(β+γ)/2					*Kontrola*		A	*(b-c)/(b+c)*
(β-γ)/2					*a*			*arctan (A * B)*